Math 32A:

Quiz: 11/17/23 (Friday)

Midterm: 11/20/23 (Monday)

OH: Thurs 2-3pm (will be online if I'm still sick)

Midterm Review: This weekend, will announce by end of day tomorrow

Reminder: Fill out TA survey by Friday!

Quiz Review

$r(t)$ parameterizing a curve C

$v(t) = r'(t)$ __velocity__ — vector that is tangent to C

$a(t) = r''(t)$ __acceleration__

Unit Tangent Vector:

$$T(t) = \frac{r'(t)}{\|r'(t)\|}$$

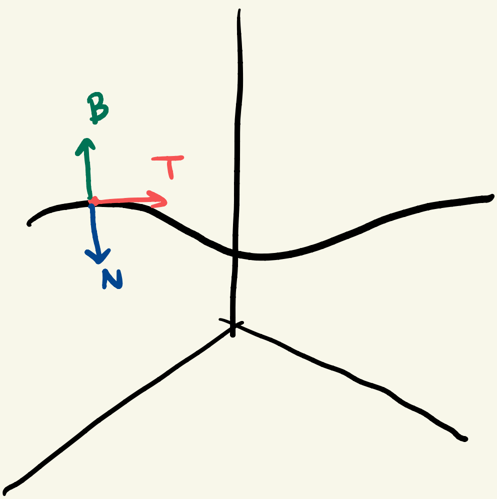
Vector of unit length $(\|T\| = 1)$

Unit Normal Vector:

$$N(t) = \frac{T'(t)}{\|T'(t)\|}$$

$T'(t)$ is always orthogonal to $T(t)$

$$\boxed{\begin{aligned}
\|T(t)\|^2 = 1^2 &= 1 \\
T(t)\cdot T(t) &= 1 \\
\frac{d}{dt}\left(T(t)\cdot T(t)\right) &= \frac{d}{dt} 1 \\
T'(t)\cdot T(t) + T(t)\cdot T'(t) &= 0 \\
\Rightarrow \quad 2\, T'(t)\cdot T(t) &= 0 \\
\Rightarrow \quad T'(t)\cdot T(t) &= 0
\end{aligned}}$$



Binormal Vector: $B(t) = T(t) \times N(t)$

$$\|B\| = \|T\|\,\|N\|\,\underbrace{\cancel{\sin\theta}}_{1} = 1$$

Curvature:

κ defined as $\left\| \frac{dT}{ds} \right\|$ ← Find arc length parametrization, then find $T(s)$, and then $\|T'(s)\|$.

$$\kappa(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$$

Acceleration Thm: $a(t) = r''(t)$

$$\left[\; a(t) = a_T T + a_N N \;\right]$$

T: unit tangent
N: unit normal
a_T, a_N components

$$a_T = a \cdot T = \frac{a \cdot v}{\|v\|}$$

$$\left[\; a_N = a \cdot N = \sqrt{\|a\|^2 - |a_T|^2} \;\right]$$

$$a_T T = \left(\frac{a \cdot v}{v \cdot v}\right) v, \quad a_N N = a - a_T T = a - \left(\frac{a \cdot v}{v \cdot v}\right) v$$

① At time $t=2$, a particle has $v(2) = \langle 1,1,1 \rangle$ and $a(2) = \langle -2,0,2 \rangle$. Compute the unit tangent and unit normal vectors at $t=2$:

$$T(t) = \frac{v(t)}{\|v(t)\|}$$

$$T(2) = \frac{v(2)}{\|v(2)\|} = \frac{1}{\sqrt{3}}\langle 1,1,1 \rangle$$

$$\cancel{N(t) = \frac{T'(t)}{\|T'(t)\|}}$$

Can't use this b/c we only have a single pt.

$$\left[a(t) = a_T T + a_N N \right]$$

$$a_N N = a - a_T T$$

$$a_T = a \cdot T = \langle -2,0,2 \rangle \cdot \frac{1}{\sqrt{3}}\langle 1,1,1 \rangle$$

$$= -\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} = 0$$

$$\Rightarrow \quad a_N N = a$$

$$\|a\| = \sqrt{4+0+4} = 2\sqrt{2}$$

$$N = \frac{a}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}\langle -2,0,2 \rangle = \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$$

(2) For any curve $r(t)$, with respect to T, N, B vectors, in which direction will $v(t) \times a(t)$ point?

(Describe the direction in terms of T, N, B vectors)

$$\left[a(t) = a_T T + a_N N \right]$$

$$T(t) = \frac{v(t)}{\|v(t)\|}$$

$$\underline{v(t) \times a(t)} = \|v(t)\| T \times (a_T T + a_N N)$$

$$= \|v(t)\| a_T \overset{0}{\cancel{T \times T}} + \|v(t)\| a_N \overset{B}{\cancel{T \times N}}$$

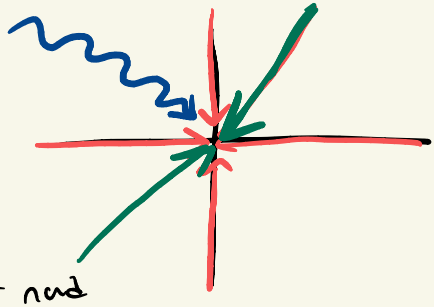
$$= \|v(t)\| a_N \underline{B}$$

In direction of Binormal vector, B.

Limits for Multivariable Functions:

① $$\lim_{(x,y)\to \underline{(0,1)}} \frac{e^x}{x-4y} = \frac{e^0}{0-4} = \boxed{-\frac{1}{4}}$$

② $$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$



To show limit DNE, just need to find two paths where limit is different

x-axis: $y=0$: $$\lim_{(x,0)\to(0,0)} \frac{0}{x^2+0} = 0$$

y-axis: $x=0$: $$\lim_{(0,y)\to(0,0)} \frac{0}{0+y^2} = 0$$

$y=x$: $$\lim_{(x,x)\to(0,0)} \frac{x^2}{x^2+x^2} = \lim_{(x,x)\to(0,0)} \frac{\cancel{x^2}}{2\cancel{x^2}} = \frac{1}{2}$$

$$\boxed{\text{Limit DNE}}$$

In general, set $y=mx$: $$\frac{x(mx)}{x^2+m^2x^2} = \frac{m\cancel{x^2}}{\cancel{x^2}(1+m^2)} = \frac{m}{1+m^2}$$

limit DNE, b/c it depends on the slope.

If it did <u>not</u> depend on m, then limit could still <u>NOT</u> exist

[ONLY A TEST TO CHECK IF SOMETHING DNE]

$f(x,y) = z$